Revisiting Thomas Carlyle and Mathematics

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Dr Brewster advises me to commence forthwith at Legendre for it will go on certainly. "I shall do it." But I have got no desk yet

Thomas Carlyle to Alexander Carlyle, 21 November 1821

In 1822, the first edition of young Thomas Carlyle's translation of the influential textbook *Eléments de géométrie* (1784), by the French mathematician Adrien-Marie Legendre (1752–1833), was published in Edinburgh by Oliver and Boyd and G. and W. B. Whittaker. Included in this translation were notes and an introductory chapter entitled "On Proportion" written by Carlyle to address the prerequisites to understanding the text that he perceived the average British student to be lacking. Although Carlisle Moore's article on Carlyle and mathematics remains extremely useful, even definitive in terms of connecting Carlyle's early mathematical acumen with his developing literary mode of thinking, some insight into the mathematics presented in Carlyle's essay deserves a revisit, especially since Carlyle's geometric approach to quadratic equations remains elegantly relevant to mathematicians today.

Prior to the publication of Legendre's *Eléments*, the primary text for the serious student of geometry was Euclid's *Elements*, which dates from about 300 BCE.[1] Legendre simplified Euclid's propositions, and the result was a more effective textbook that soon became a standard and later a prototype in Europe and the United States and replaced Euclid's as the primary geometry reference work for about the next hundred years. Carlyle's translation of the *Eléments* alone ran to 33 editions.

Since Carlyle wrote "On Proportion" to help the British student in preparation for Legendre's *Eléments*, it makes sense to review the state of British mathematics and the state of mathematics as a whole at the beginning of the nineteenth century. Near the end of the seventeenth century, the important discovery of the calculus was made independently by Isaac Newton (1642–1727) and by Gottfried Leibniz (1646–1716).[2] Their discovery provided a way for mathematicians to calculate using infinite processes, about which the ancients had known but had avoided. Because Leibniz was more public than Newton about his work, the ideas of the calculus spread more rapidly on the European continent and were furthered by the brilliant Swiss mathematician Leonhard Euler (1707–83). With this powerful new mathematical tool, Euler and others began to solve a large number of previously insoluble problems. They had particular success when they separated the calculus from the geometric context in which Newton and Leibniz developed their ideas. This new line of thinking led to the development of the mathematical subject of analytic algebra. Eventually, geometry was ostracized by the mathematicians on the European continent until the work of Gaspard Monge (1746–1818) toward the end of the eighteenth century.

The less forthcoming Newton eventually grew jealous of the fame proffered to Leibniz for his work on the calculus. After an initial period of acknowledging his German rival's contribution as significant, a bitter dispute evolved from the charge that Leibniz had plagiarized from Newton. The controversy lasts to the present day, although the current view is that it is extremely unlikely that Leibniz would have had access to Newton's work. At the time, however, the controversy caused a deep and abiding rift between the mathematicians of Britain and of the Continent. To further complicate matters, in Britain, Newton's ideas were not universally accepted by the mathematical community. Even as late as 1734, the eminent British metaphysician Bishop George Berkeley (1685–1753) very convincingly attacked the fundamental ideas of Newton's "Method of Fluxions." In response to Berkeley's attack, Newton's friend Colin Maclaurin (1698–1746), mathematics professor at the University of Edinburgh, wrote a systematic, two-volume defense, *A Treatise on Fluxions* (1742), which sought rigorously to elevate Newton's ideas to the heights of the timeless Euclid, whom Carlisle Moore correctly refers to as "the symbol of imperishable truth" (63). But Maclaurin's geometric approach did not gain the popular acceptance that the brand of analysis being developed on the Continent garnered, and it is perhaps not a coincidence that Maclaurin was practically the last significant mathematician in Britain for nearly a century.[3] And from 1730 to 1820, there was very little progress made by British mathematicians as they adopted in much of their research the ancient geometric methods delineated by Maclaurin in his *Treatise on Fluxions* and fell further behind the accomplishments of the continental mathematicians. Perhaps the only other prominent British mathematician during this time period was Matthew Stewart (1717–85), a student of Maclaurin's at Edinburgh who had succeeded him in the chair of mathematics at the university. Unfortunately, Stewart continued to concentrate on the ancient geometrical methods that were so popular at the time in Britain, and he thereby continued to ignore the analytic algebra that by then dominated mathematics on the Continent.[4]

This brief history serves to contextualize the mathematical tradition in which Carlyle and his fellow students were educated when attending classes at the University of Edinburgh taught by John Leslie (1766–1832), a mathematician and physicist most often remembered for his work on heat and capillary action. Like Maclaurin, Leslie in his *Elements of Geometry, Geometrical Analysis, and Plane Trigonometry* (1809) clung to synthetic geometry as a more intellectual alternative to analytic algebra: "It is a matter of deep regret, that Algebra, or the Modern Analysis, from the mechanical facility of its operations, has contributed, especially on the Continent, to vitiate the taste and destroy the proper relish for the strictness and purity so conspicuous in the ancient method of demonstration. The study of geometrical analysis appears admirably fitted to improve the intellect, by training it to habits of precision, arrangement, and close application" (viii–ix). That the professor imbued his student with a general passion for math and the supremacy of geometry was suggested in 1866 by Carlyle himself: "Perhaps it was mainly by accident that poor Leslie, alone of my professors, had some genius in his business, and awoke a certain enthusiasm in me. For several years, from 1813 onwards (perhaps 7 in *all*), 'Geometry' shone before me as undoubtedly the *noblest* of all sciences" (*Two Reminiscences* 36).

Carlyle's passion for geometry would yield palpable results. A reference to Carlyle in Leslie's third edition attributes to Carlyle, whom he calls an "ingenious young mathematician, formerly my pupil" (340), an alternate compass and straight edge construction to the one proposed by Pappus of Alexandria in about 320 CE (176–77). Carlyle's solution is based upon the premise that the lengths of the sides of a rectangle are uniquely determined if one knows the product of these lengths (i.e., the area of the rectangle) and the sum of these lengths (i.e., one-half the perimeter of the rectangle). In 1991 Duane W. DeTemple provided an equivalent formulation for the solution based upon what he calls "Carlyle Circles" (99–100). DeTemple refers to Carlyle's geometric solution to quadratic equations as "elegant" and "especially attractive" (99), comments that suggest a rather suprising relevance.

In Carlyle's introductory essay, "On Proportion," it is clear from the first paragraph that he has been exposed to the strengths and the weaknesses in the approaches intended to extend the understanding of the ancients: "Owing, however, to our general and long-continued employment of Euclid's Elements, the fifth Book of which is devoted to proportion, our common systems of mathematics, and even of Algebra, pass over the subject in silence, or allude to it so slightly as to provide no adequate information" (ix). If Carlyle was potentially a defender of geometry, Legendre was one of the major figures in the development of analytic algebra on the European Continent, although at the same time he was well-known for his work on the attractions of ellipsoids studied by Maclaurin. Ultimately, it is also true that in writing *Eléments*, Legendre was trying to revive the intellectual quality of Euclid that had been surpassed by the rapid advances via analysis that had occurred on the Continent. Carlyle, who had studied Newton's work extensively while a student at Edinburgh, would have been familiar with the basic techniques derived from the calculus that Legendre employed in his text.[5] Therefore, given the British tradition that Carlyle was educated in, his familiarity with the calculus, and his work on geometric solutions to quadratic equations, Legendre's *Eléments* was a very appropriate text for Carlyle to translate.

In "On Proportion" there is very little in the way of original mathematics, although this lack is not surprising given that the essay is an introduction to an introductory geometry textbook. What is interesting about "On Proportion" is Carlyle's use of the ideas of the calculus to extend the classical results on proportion to incommensurable magnitudes. Two magnitudes or lengths a and b are "commensurable" if their ratio a/b is a rational number, hence there exist two integers p and q such that $a/b = p/q$. If p and q are the smallest such positive integers, then the common measure of a and b is $a/p = b/q$. The two magnitudes or lengths a and b are "incommensurable" if a/b is not rational.

Although incommensurable magnitudes or lengths are easy to construct (the length of the side of a square is incommensurable with the length of the diagonal of the same square, an example that Carlyle provides in a footnote [xi]), the ancients were uncomfortable with incommensurable magnitudes and tended to avoid them whenever possible. In an understandable if self-serving moment, Carlyle articulates the ancients' abandonment of incommensurable magnitudes:

> The proper mode of treating proportion has given rise to much controversy among mathematicians; chiefly originating from the difficulties which occur in the application of theorems to that class of magnitudes denominated *incommensurable*, or having no *common measure*. Euclid evades this obstacle; but his method is cumbrous, and, to a learner, difficult of comprehension. All other methods have the disadvantage of frequently employing the principle of *reductio ad absurdum*, a species of reasoning, which, though perfectly conclusive, the mathematician wants to employ as seldom as possible. (ix)

In current mathematical literature, this technique is referred to as "proof by contradiction." The reason such proofs should be avoided is that they tend to be "non-constructive." Typically, one assumes the opposite of what is to be proved and then establishes a logical contradiction that shows this cannot be the case. For example, we refer the reader to the standard proof that $\sqrt{2}$ is not a rational number.[6] Although the proof is logically valid, it gives us no idea what the numerical value of $\sqrt{2}$ might be. Carlyle continues by pointing out that the practical results yielded by the *reductio ad absurdum* approach had led to Euclid's method being abandoned, and also that "[o]n this matter we are happily delivered from the necessity of making any selection; the author having himself provided for the application of proportion to incommensurable quantities, and demonstrated every case of this kind as it occurred, by means of the *reductio ad absurdum*" (ix).

In "On Proportion" Carlyle follows Legendre's example of working with incommensurable magnitudes by applying the ideas of the calculus. This approach allows one to be constructive by using the "limit process" that serves as the fundamental idea of the calculus. The limit process allows one to calculate with an infinite sequence of successively better and better approximations to a quantity instead of with the quantity itself. Carlyle relegates this process to the lengthy footnotes given on pages xi and xiii of his introductory essay, which allows the reader to concentrate on the fundamental ideas of the relations that exist between commensurable magnitudes while giving the interested reader a constructive way to address the complications involved with incommensurable magnitudes. In particular, Carlyle tells the reader in the last sentence of the footnote on page xi that "our Definition includes incommensurable as well as commensurable quantities; and whatever is found to be true of proportions among the latter, may also, by the method of *reductio ad absurdum*, be shewn to hold good when applied to the latter [sic; former]" (ix). Although Carlyle is still using the method of *reductio ad absurdum*, because of the limit process, the proofs he outlines here are indeed more constructive than most proofs using this method. As a result, these proofs are more advantageous to the student of Legendre.

Once the technique for working with incommensurable quantities is established, Carlyle then goes on to prove a classical theorem which he calls Theorem I. This proof confirms the central theorem of the essay as it allows one to reduce the statement of four magnitudes being proportional to a statement about certain products of these magnitudes being equal. Once he applies this approach for incommensurable quantities as well as commensurable quantities, the rest of the essay can be confined to statements about the multiplicative properties of the real numbers and easily follows from Theorem I. The advantage of this presentation to the reader is that it provides the student with a useful list of the facts—with proof—that Legendre later assumes in his text. In "On Proportion" and in this translation of Legendre's *Eléments* we see the beginnings of a more fluent exchange of ideas between the British and the Continental mathematical communities. Carlyle acknowledges this purpose in his introductory paragraph. He is, after all, trying to make this product of Continental mathematical thought more accessible to the British student. The overwhelming success of this work signifies a wider acceptance of the mathematics developed on the European continent as well as the incorporation of the calculus at a more foundational level.

In *Reminiscences*, Carlyle remembered his work on Legendre: "I still remember a happy forenoon (Sunday, I fear!) in which I did a *Fifth Book* (or 'Complete Doctrine of Proportion') for that work; complete really and lucid, and yet one of the *briefest* ever known; it was begun and done that forenoon, and I have (except correcting the press next week) never seen it since, but still feel as if it were right enough and felicitous in its kind! I got only £50 for my entire trouble in that *Legendre*, and had already ceased to be in the least proud of *Mathematical* prowess; but it was an honest job of work honestly done" (271–72). Carlyle would lose his interest in mathematics, but his early work on the subject, including that associated with "Carlyle Circles" and Legendre, continues to exert a "felicitous" influence on a field that Carlyle would soon abandon but not forget as he turned his intellect towards literature.

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Notes

1. See Carl B. Boyer and Uta C. Merzbach, *A History of Mathematics*, 100–119.

2. Although Isaac Newton was the first to discover the calculus (the first account of which was *De analysi per aequationes numero terminorum infinitas*, composed in 1669 but not published until 1711), Newton did not present his results to the general mathematics community until 1687 when *Philosophiae naturalis principia mathematica* appeared. In the meantime, Gottfried Leibniz had published in 1684 the first paper on differential calculus entitled *Nova methodus pro maximis et minimis, itemque tangentibus, qua nec irrationales quantitates moratur.* Two years later, Leibniz published an explanation of the integral calculus.

3. See Boyer and Merzbach, 430.

4. See Florian Cajori, *History of Mathematics*, 277.

5. See Carlisle Moore 63–64. For examples of TC's exposure to Newton and others, see also *CLO*: TC to Robert Mitchell, 16 July 1816, 5 July 1817, and 25 May 1818.

6. See John Beachy and William Blair (xix). In addition, the proof for $\sqrt{2}$ as irrational is available in most introductory abstract algebra textbooks and is based upon the conclusion that if $\sqrt{2} = a/b$, where both a and b are whole numbers whose greatest common divisor is 1, and b is not 0, it can be shown that both a and b are divisible by 2 using the observation that $a^2 = 2b^2$. This contradicts the original assumption that 2 is not a common divisor of a and b. Therefore, $\sqrt{2}$ must be an irrational number.

Works Cited

Beachy, John, and William Blair. *Abstract Algebra.* 3rd ed. Long Grove, IL: Waveland Press, 2006.

Boyer, Carl B., and Uta C. Merzbach. *A History of Mathematics.* 2nd ed. New York: Wiley, 1991.

Cajori, Florian. *A History of Mathematics.* 3rd ed. New York: Chelsea, 1980.

Carlyle, Thomas. "On Proportion." *Elements of Geometry and Trigonometry* [by A. M. Legendre]. Ed. David Brewster. Edinburgh, Scot.: Oliver and Boyd, 1822. ix–xvi.

———. *Reminiscences*. Ed. K. J. Fielding and Ian Campbell. Oxford, Eng.: Oxford UP, 1997.

———. *Two Reminiscences of Thomas Carlyle Now First Published*. Ed. John Clubbe. Durham, NC, Duke UP, 1974.

Carlyle, Thomas, and Jane Welsh Carlyle. *The Carlyle Letters Online* (*CLO*). Coordinating Ed. Brent E. Kinser. Durham, NC: Duke UP, 2007. Web. 15 Nov. 2008.

DeTemple, Duane. "Carlyle Circles and the Lemoine Simplicity of Polygon Constructions," *American Mathematical Monthly* 98.2 (1991): 97–108.

Leslie, John. *Elements of Geometry and Plane Trigonometry*. Edinburgh, Scot.: Brown and Crombie, John Ballantyne; London: Longman, Hurst, Rees, Orme, Murray, Scholey, Mawman; Cambridge, Eng.: Deighton, 1809.

———. *Elements of Geometry and Plane Trigonometry*. 3rd ed. Edinburgh, Scot.: Constable; London: Longman, Hurt, Rees, Orme, and Brown, 1817.

Moore, Carlisle. "Carlyle: Mathematics and 'Mathesis.'" *Carlyle Past and Present*. Ed. Kenneth J. Fielding and Rodger L. Tarr. London: Vision Press, 1976.